

International Conference on Industrial Engineering, ICIE 2016

Theoretical Study of the Load Distribution on the Threads for Roller Screw Mechanisms of a Friction Type

A. V. Zhdanov^a, V. V. Morozov^{a,*}

^a *Vladimir State University named after Alexander and Nikolay Stoletovs, Gorkogo Street 87, Vladimir, 600000, Russia*

Abstract

The paper provides theoretical research of load distribution across the thread turns of the planetary roller-screw mechanism (RSM). A characteristic feature of the theoretical approach of the paper is the use of a RSM rod model with two elastic contact layers (the first layer is composed of screw-rollers, the second – of nut-rollers). A threaded mating of each elastic layer is considered as a connection with continuous turns, which is closest to the truth and consistent to classical approaches. A similar approach was applied for the ball-and-screw mechanisms with a member point contact. Thus, differential analytical closed-form equations may be used to solve this task.

Based on the initial equation of compatibility of strains, axial offsets of contact layer points and the sum of screw, roller and nut turn deflections are calculated; then, load distribution across the turns may be determined. Load distribution depends both on the RSM geometry (pitch, profile angle, entry number, and thread diameter), material of threaded elements, manufacturing accuracy, and on mating friction forces. With the dependencies of load distribution across the turns of zero-clearance RSMs obtained, we have determined that maximum load is imposed on the outermost turns, which corresponds to classical solutions proposed by N. E. Zhukovski.

© 2016 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

Peer-review under responsibility of the organizing committee of ICIE 2016

Keywords: distribution; Roller Screw Mechanisms (RSM); elastic contact layers

1. Introduction

Roller screw mechanisms (RSM) with a planetary motion of the threaded rollers are rolling transmissions and provide forward movement of the output link. They successfully compete with gears screw-nut ball screws and

* Corresponding author. Tel.: +7-915-768-49-68
E-mail address: zhdanov@vlsu.ru

sliding mechanisms [1-5]. Durability, reliability and durability PBS determined stress levels and efforts in the elements, which depends on the material characteristics, the geometric parameters of the contacting surfaces and load distribution law for the threads [6-11].

The behaviour of the load distribution between the thread coils of the planetary RSM is necessary for the calculation of their load capacity, stiffness, durability and other force parameters. The classical solution for the task of the load distribution between the thread coils was found by N.E. Zhukovsky [12]. He thought the conjugation of screw-nut as a discrete model, based on which is solved the finite-differential equation. The detailed studies of the question about the load distribution along the thread coils of the transmission screw-nut and the ball-screws mechanisms (BSM) sliding were carried out in the works of I. A. Birger and G. B. Iosilevich [12]. They examined the threaded coupling as a connection whit continuous running coils that is closest to the reality. This made it possible to apply to this problem differential equations whose solution is obtained in closed-form. Later G. B Iosilevich developed the most common approach, which is based on the concept of conditional contact layer, which significantly simplifies the conclusion of the calculated dependencies [13]. The coefficients for load distribution on the threads RSM proposed in [14-19]. The catalogs RSM [20] this issue is not addressed. Recently, however, a number of works devoted to the issue of the study load distribution on the threads [14-21], in which the analytical and numerical solutions. In this paper, the theoretical study of load distribution tasks to the threads RVM.

A RSM has two spatial couplings: screw-rollers and rollers-nut, which separately must solve the problem of N.E. Zhukovsky. However, the geometric parameters (screw profile, point contact, mixed friction in the connection) should impose its own characteristics on the solution. During the calculations design, and the studies of static and dynamic of the load-carrying capacity, the irregularity of the load distribution was estimated by the coefficient k_n , which does not considerate the change in the actual geometry of the profiles due to the manufacturing errors and deterioration, as well as the influence of the friction forces in the connection. The question of the load distribution along the thread of the RSM thread was answered by V.V. Kozyrev [22], which obtained the following expression for the calculation of the design:

$$k = \left(\left(\frac{1}{k_{rg}} + \frac{1}{k_{rm}} + \frac{1}{k_{rn}} \right) - 2 \right)^{-1} \quad (1)$$

where k_{rg} , k_{rm} , k_{rn} - coefficients of the the irregularities depending on the stiffness of the transmission parts, the upsetting moment in the plane of the roller axis and the manufacturing errors of the thread, respectively. These coefficients are obtained from the known dependencies of the BSM and do not considerate the structural and geometric features of the RSM.

2. Methods and materials

The problem of the load distribution along the thread coils of the PBM consider a model with two elastic contact layers. The Fig. 1 and shows a planetary RSM. The Figure 1 b shows the calculated the output scheme of the equation of the load distribution on the thread coils. On the Figure 1, b are indicated: 1 - screw; 2 and 3 elastic contact layers, simulating the contact in the coupling screw-roller or roller-nut and body-roll; 4 – nut.

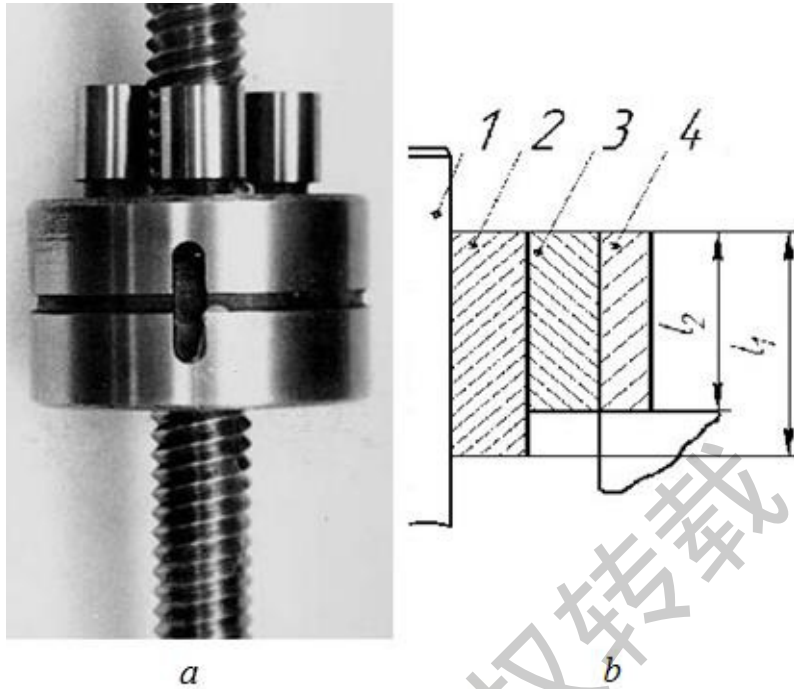


Fig. 1. Planetary RSM (a) RSM for a medical drive function (b) design scheme

The compatibility equation of deformation in the RSM is given similarly to the expressions in the paper:

$$\Delta_I(z_1) - \Delta_{II}(z_3) = \delta_1(z_1) + \delta_2(z_1) + \delta_2'(z_3) + \delta_3(z_3) - [\delta_1(0) + \delta_2(0) + \delta_2'(0) + \delta_3(0)] \quad (2)$$

where $\Delta_I(z_1)$ - is the axial displacement of the points of the first contact layer, which is determined by the coordinate z_1 ; $\Delta_{II}(z_3)$ is the axial displacement of the points of the second contact layer, which is determined by the coordinate z_3 ; $\delta_i(z_i)$ is the amount of deflection of the screw coils, rollers or nuts, and deformation of the contact in the axial direction in the section z_i ; $\delta_i(0)$ - also in the section z_i ; the index $i=1,2$, and 3 correspond to the screw rolls and nut, respectively.

The external axial force Q , acting on the RSM, is distributed between two contacting layers: screw-rollers (I) and rollers-nut (II). The magnitude of axial load on each layer is given by:

$$Q_j = \int_0^{z_i} q_j(z_i) dz_i \quad (3)$$

where z_i - is the length i - of connection, q_i - is the basic axial effort according to the thread height of the i -th element.

For the rod of the RSM model, represented on the Fig. 1b, the axial displacement can be presented as:

$$\Delta_I(z_1) = \int_0^{z_1} \frac{Q_I(z_1)}{E_I A_I} dz_1, -\Delta_{II}(z_3) = \int_0^{z_3} \frac{Q_{II}(z_3)}{E_{II} A_{II}} dz_3 \quad (4)$$

where $1/E_1 A_1$ is the compliance section of the j -th bar (elastic layer), at tension-compression ($j=I, II$); Q_j - is the axial force acting in the section of the z_j j -th bar..

The axial movement δ_i of the coils of the i -th element can be represented as a function of:

$$\delta_i = \left[\lambda_i^* \frac{P_{n_i}}{E_i} + \nu_i^* \sqrt{\left(\frac{S_{ip} P_{n_i}}{E_i} \right)^2 \frac{1}{\rho_{(1)}}} \right] \cos \Psi_i \quad (5)$$

where S_{ip} -is the distance between the elementary connections along the contact line; $\rho_{(1)}$ -is the reduced radius of the curvature of the i -th connection in the cross-section conjugation; Ψ_i -is the angle inclination of the screw line

3. Results

The equation (5) takes into account the geometric features of the RSM in terms of the magnitudes S_{ip} , $\rho_{(1)}$ and Ψ_i . The distortion of the roller in the model shown in the Figure 2a can be considered as a relative displacement of the elastic layer I with respect to the elastic layer II.

To determine the influence of the friction force in the thread connection on the load distribution along the coils, we consider the equation of equilibrium of the i -th element of the RSM in the traction mode of the straight lead. This equation can be written as:

$$p_{n_i} \sin \alpha \cos \Psi_i = q_i + f_{p_i} \quad (6)$$

where p_{n_i} -is the normal effort in the gearing elements oh the RSM; α - is the angle of the thread profile of the i -th element in the axial section; f_{p_i} -is the elementary vertical component of the frictional force applied to the i -th element.

The average effort in the gearings is defined as

$$p_{n_i} = \frac{q_i + f_{p_i}}{\sin \alpha \cos \Psi_i} = \frac{q_i}{\sin \alpha \cos \Psi_i (1 \pm k_{12(23)})} \quad (7)$$

where $k_{12(23)}$ -is the influence coefficients of the external friction force on the face of the plane:

$$k_{12} = \frac{f_{12} \sin \theta}{\cos \gamma}, \quad -k_{23} = \frac{k_{23} \sin \theta}{\cos \gamma} \quad (8)$$

where $\theta = \arctg(\tg \delta \sin \delta)$ is the angle between the direction of the frictional force and the face of the plane; $f_{12(23)}$ is the friction coefficient between the elements 1-2 and 2-3 respectively; δ is the angle between the friction force and the line of intersection of the tangential and facial plane.

The intensity distribution of loads $q_j(i)$ and efforts p_{n_i} associated to the screw parameter $p_b = p_x / L = p_x / D_{ik} \pi$:

$$p_{n_i} = \frac{q_i + f_{p_i}}{\sin \alpha \cos \Psi_i} = \frac{q_i}{\sin \alpha \cos \Psi_i (1 \pm k_{12(23)}) \pi D_{ik}} \quad (9)$$

where p_x -is the helix pitch; D_{ik} -is the diameter of the cylindrical surface on which the contact point is distributed.

Substituting (5) and (9) into equation of the compatibility of deformations, we obtain the equation (2) in the form:

$$\frac{1}{E_I A_I} \int_0^{z_1} Q_I(z_1) dz_1 - \frac{1}{E_{II} A_{II}} \int_0^{z_3} Q_{II}(z_3) dz_3 = q_I(z_1) \gamma_I + q_I^{2/3}(z_1) \gamma_{kI} + q_{II}(z_3) \gamma_{II} + q_{II}^{2/3}(z_3) \gamma_{kII} - c \quad (10)$$

where $c = q_I(0) \gamma_I + q_I^{2/3}(0) \gamma_{kI} + q_{II}(0) \gamma_{II} + q_{II}^{2/3}(0) \gamma_{kII}$

To simplify the resulting expression we use the relation:

$$q^{2/3} = a + bq \quad (11)$$

where a and b- are constant coefficients.

The magnitude a does not affect the load distribution, the factor b is found from the condition, which is approximately straight and is a tangent at $q=q_p$. For these reason

$$b_j = \frac{2}{3} \frac{1}{\sqrt[3]{q_{-pj}}}, \quad q_{-pj} = \frac{Q_j}{z_i} \quad (12)$$

Making these assumptions allows us to consider the contact layer linearly as elastic; for which we obtain the relation:

$$d\delta_i(z_i) = \lambda_{mi} dq_i(z_i) \quad (13)$$

From the conditions $Q_I(z_1) = Q_{II}(nz_3)$ and relations $q_i = dQ_i(z_i)/dz_i$ we obtain the expressions for the derivatives:

$$q_I(z_1) = nq_{II}(nz_3), \quad q_I'(z_1) = n^2 q_{II}'(nz_3), \quad q_I''(z_1) = n^3 q_{II}''(nz_3) \quad (14)$$

where $n = z_3/z_1$;

The given expressions (13) - (14), and the equation (10) can be rewritten as:

$$\frac{Q_I(z_1)}{E_I A_I} dz_1 + \frac{Q_{II}(z_3)}{E_{II} A_{II}} dz_3 = \lambda_{m1} q_I^{2/3}(z_1) dz_1 + \lambda_{m2} q_{II}^{2/3}(z_3) dz_3 \quad (15)$$

or

$$\frac{Q_I(z_1)}{E_I A_I} + \frac{Q_{II}(z_3)}{E_{II} A_{II}} = \lambda_{m1} q_I^{2/3}(z_1) + \lambda_{m2} q_{II}^{2/3}(z_3) dz_3 \quad (16)$$

Re-differentiating (65) and substituting the relation (14), we obtain the linear differential equation of the second order:

$$q_I''(z_1) - \beta_p^2 q_I(z_1) = 0 \quad (17)$$

where

$$\beta_p = \sqrt{\frac{1 + \frac{n}{E_I A_I + E_{II} A_{II}}}{\lambda_{m1} + \frac{\lambda_{m2}}{n}}} = \sqrt{\frac{\alpha}{\lambda_q}} \quad (18)$$

The solution of the equation (17) can be written as

$$q_I(z_1) = Ach\beta_p z_1 + Bsh\beta_p z_1$$

where the constants A and B are determined from the boundary conditions given in [9].

With this in mind, we obtain the load on the i-th coil:

$$q_i(z_i) = \frac{Q\beta_p}{\alpha sh\beta_p l_i} \left[\frac{ch\beta_p z_i}{E_I A_I} + \frac{nch\beta_p (l_i - z_i)}{E_{II} A_{II}} \right] \quad (19)$$

where l_i - is the length of the contact layer part.

For the calculation it is necessary to know the elastic and geometric parameters of the contact layers: λ_{m1} , λ_{m2} , β_p . After the transformations we obtain

$$\lambda_{m1} = \gamma_I + b_I \gamma_{kI}, \quad \lambda_{m2} = \gamma_{II} + b_{II} \gamma_{kII} \quad (20)$$

where

$$\gamma_I = \frac{P_x}{\pi \sin \alpha (1 \pm k_{12})} \sum_{i=1}^2 \left(\frac{\lambda_i^*}{D_{ik} E_i} \right),$$

$$\gamma_{II} = \frac{P_x}{\pi \sin \alpha (1 \pm k_{23})} \sum_{i=2}^3 \left(\frac{\lambda_i^*}{D_{ik} E_i} \right),$$

$$\gamma_{kI} = \sqrt[3]{\left(\frac{P_x}{\pi \sin \alpha (1 \pm k_{12})} \right)^2 \frac{1}{\rho_{(1)}} \sum_{i=1}^2 \left[v_i^* \sqrt[3]{\left(\frac{S_{ip}}{D_{ik} E_i} \right)^2 \cos \Psi_i} \right]}$$

$$\gamma_{kII} = \sqrt[3]{\left(\frac{P_x}{\pi \sin \alpha (1 \pm k_{23})} \right)^2 \frac{1}{\rho_{(1)}} \sum_{i=2}^3 \left[v_i^* \sqrt[3]{\left(\frac{S_{ip}}{D_{ik} E_i} \right)^2 \cos \Psi_i} \right]}$$

To determine β_p we use the formula (18) and it does not cause difficulties. To identify γ_{ki} and λ_{mi} it is necessary to know the magnitudes of λ_i^* and v_i^* , which are obtained by solving the problem of the elastic deformation of the contact layer.

The coefficients can be found theoretically:

$$v_i^* = n_\delta \sin \alpha_k S_p^{2/3},$$

$$\lambda_i^* = ctg\alpha_k \left[2k \sum_i (k_{1i} + k_{2i} + k_{3i}) + \frac{1-\nu^2}{2\pi} \left(\sum_i L_i(l) + \sum_i \delta(y)\chi(x) \right) \right]$$

The Fig. 2 shows the results of the theoretical calculation of the load distribution on the coils of the RSM for the connections I and II. The absolute value of the load for the transmissions made from the same material increases with a reduced modulus of elasticity E in the connection, reaching Fmax=1300 H at E=1•105 MPa and decreasing to Fmax=1050 H at E=2,1•105 MPa. For the transmissions, in which the elements are made from different materials (Fig.3.10), the load on the first coil is maximal, if the reduced modulus of elasticity of the connection screw-rollers is bigger than the modulus of elasticity of the connection roller-nut (Esr≥Ern) and the load is maximal on the last coil if Esr≤Ern. With a coefficient of friction in the connection f=0,1 the maximal load Fmax on the most heavily loaded coil is 1,67 of the average load Fmid in the connection Fmid; at f=0,15-Fmax=1,57Fmid; at f=0,2-Fmax=1,48Fmid.

At the load on the first and last coils are close according to the value (F1=1100 H, F36=1000 H); if Asr≥Arn, then F1=1300 H, F36=950 H; at Asr≤Arn, F1=950 H, F36=1150 H.

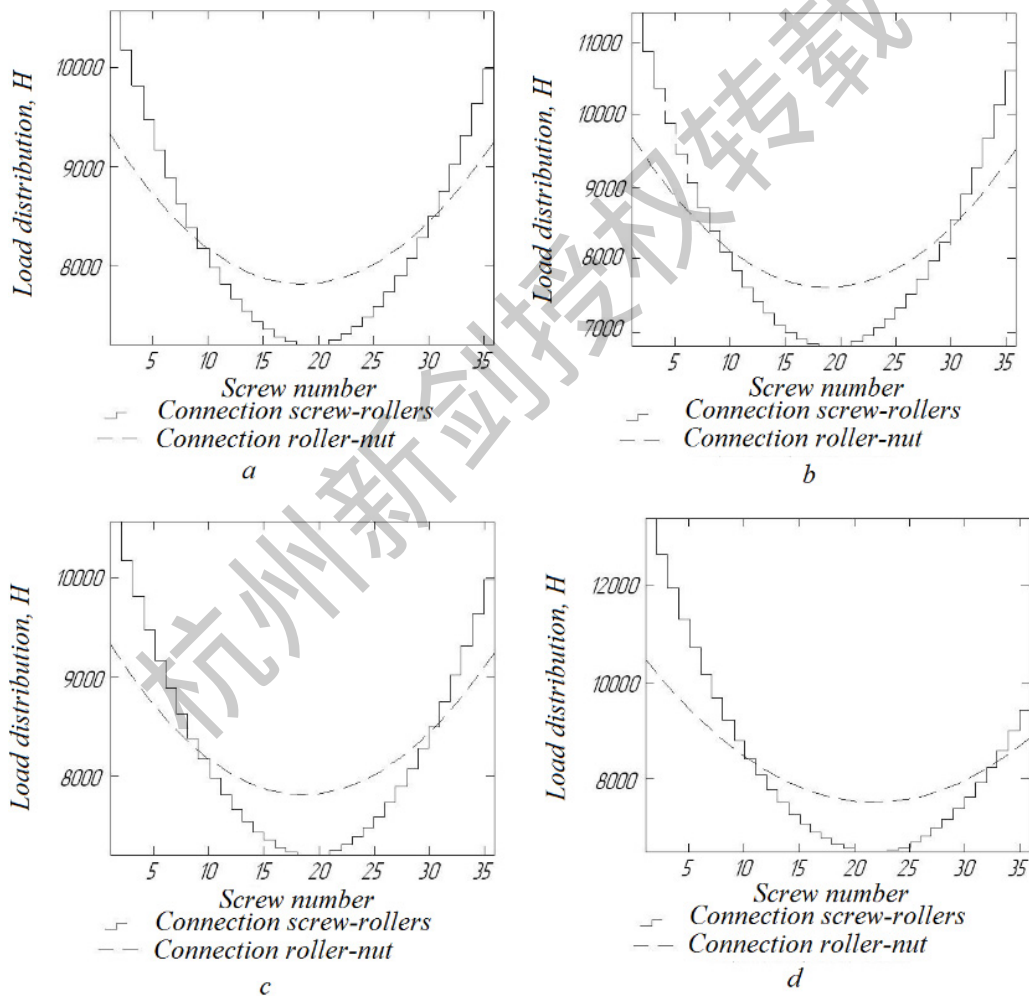


Fig. 2. Screw-rollers characteristic

References

- [1] V.V. Kozyrev, Comparison of ball and roller screw-nut transmissions, *Bulletin of Mechanical Engineering*. 11 (1983) 31–35.
- [2] V.V. Morozov, *Roller screw mechanisms, Kinematic Characteristics: Monograph*, Vladimir State University, Vladimir, 2005.
- [3] V.V. Kozyrev, *Design theory and methods of design and study of planetary gear-nut threaded rollers and mechatronic units based on them: monograph*, Vladimir State University, Vladimir, 2011.
- [4] D.S. Blinov, *Planetary roller screw mechanisms: design and calculation methods: monograph*, Bauman Moscow state technical university, Moscow, 2006.
- [5] D.V. Bushenin, V.V. Morozov, S. P. Nosatov, B.K. Popov, *Design of screw mechanisms*, Vladimir State University, Vladimir, 1982.
- [6] V.V. Morozov, A.V. Zhdanov, Influence of the axial angle of screw profiles on the load capacity of roller–screw mechanisms, *Russian Engineering Research*. 35 (2015) 477–480.
- [7] V. Morozov, A. Zhdanov, New Aspects of the Planetary Roller-screw Mechanism Classification, in: *Proceeding of International Conference on Automation, Mechanical and Electrical Engineering*, Phuket. (2015) 875–881.
- [8] A.V. Zhdanov, *Increased reliability and durability of roller screw mechanisms*, Ph.D. diss., Vladimir, 1998.
- [9] V.V. Kozyrev, *Design theory and methods of design and study of planetary gear-nut threaded rollers and mechatronic units based on them: monograph*, Vladimir State University, Vladimir, 2011.
- [10] P.A. Sokolov, O.A. Ryakhovskii, D.S. Blinov, A. Laptev, Kinematics of planetary roller screw mechanisms, *Vestnik Mashinostroeniya*. 1 (2005) 3–14.
- [11] D.S. Blinov, M.I. Morozov Promising designs of planetary roller screw mechanisms, *Izvestiya vuzov, Mashinostroenie*. 3 (2013) 62–72.
- [12] N.E. Zhukovskiy, *Selected works*, GRFML, Moscow, 1912.
- [13] I.A. Birger, G.B. Iosilevich, *Threaded connections (Library constructor)*, Mechanical Engineering, Moscow, 1973.
- [14] G. Liu, R. Tong, X. Zhang, New Study on the Parameter Relationships of Planetary Roller Screws, *Mathematical Problems in Engineering*. (2012). DOI: 10.1155/2012/340437.
- [15] J. Ris, F. Lisowski, The computational model of the load distribution between elements in planetary roller screw, in: *Proceeding of 9th International Conference on Fracture and Strength of Solids*. (2013).
- [16] M.H. Jones, S.A. Velinsky, Kinematics of roller migration in the planetary roller screw mechanism, *Journal of Mechanical Design*. 6 (2002).
- [17] S.A. Velinsky, B. Chu, T.A. Lasky, Kinematics and efficiency analysis of the planetary roller screw mechanism, *Journal of Mechanical Design*. 1 (2009).
- [18] S. Ma, G. Liu, J. Zhou, R. Tong, Optimal design and contact analysis for Planetary Roller Screw, *Applied Mechanics and Materials*. 86 (2011) 361–364.
- [19] X. Zhang, G. Liu, S. Ma, R. Tong, H. Luo, Study on axial contact deformation of planetary roller screw, *Applied Mechanics and Materials*. 155–156 (2012) 779–783.
- [20] SKF Roller screws: catalogue, SKF Group, 2014.
- [21] A. Zhdanov, AV Stepenkin, AZ Shalamberidze, Theoretical study of the load distribution on the threads RVM, *Nnauchno Technical Gazette Volga*. 2 (2011) 87–90.
- [22] G.B. Iosilevich, Concentration of stress and deformation in the machine parts, *Mechanical Engineering*, Moscow, 1981.